3 AVERAGE

> ***Do You know ?***
> Average is always greater than the least value and lesser than the highest value.

Simple Average (or Mean) is defined as the ratio of sum of the quantities to the number of quantities.

By Def ^{n}, Average $= \dfrac{\text{Sum of all quantities}}{\text{no. of quantities}}$

Putting in symbols, $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

Where $\Sigma x_i = x_1 + x_2 + x_3 +$ -----------$+ x_n$

Here $x_1, x_2, x_3,$ ----------- x_n represent the n values of quantity under consideration & $\bar{x}$ is the mean. Average or mean is said to be a measure of central tendency. It is a statistical tool that gives us the central value around which the data under consideration is distributed.

Let us take a very simple example of the first five natural numbers 1, 2, 3, 4 & 5.

Their Average = Sum/5 =15/5 = 3.

Now let's add 2 more 3's to these 5 numbers.

Now, Average = $\dfrac{15+6}{5+2} = \dfrac{21}{7} = 3.$

This can also be calculated as $\frac{1}{7} \times 1 + \frac{1}{7} \times 2 + \frac{3}{7} \times 3 + \frac{1}{7} \times 4 + \frac{1}{7} \times 5$, where $\frac{3}{7}$ & $\frac{1}{7}$ are the weights associated with the number 3 and the remaining 4 numbers. Weight associated with a particular observation depends upon its frequency (the number of times it occurs).

Weighted Mean

In the calculation of simple average each item of the series is considered equally important but there may be cases where all items may not have equal importance and some of them may be comparatively more important than others (as in the above example, 3 has thrice as much importance or weightage than each of the other numbers.) In such cases, proper weightage is to be given to various items – the weights attached to each item being proportional to the importance of the item in the distribution.

> ***Do You know ?***
> Mean has a tendency to tilt towards the side with the greatest no of entries.
> $\overline{x}$ cannot be greater than the greatest no. or less than the least no.

Let $W_1, W_2, W_3, \ldots\ldots W_n$ be the weights attached to variable values $X_1, X_2, X_3, \ldots\ldots X_n$ respectively.

Then the weighted arithmetic mean, usually denoted by

$$\overline{X}_W = \frac{W_1X_1 + W_2X_2 + W_3X_3 + \ldots.. + W_nX_n}{W_1 + W_2 + W_3 + \ldots.. + W_n} = \frac{\sum WX}{\sum W}$$

where, $W_1, W_2, W_3, \ldots\ldots W_n$ are the respective weights of $X_1, X_2, X_3, \ldots\ldots X_n$

In case of frequency distribution, $f_1, f_2, f_3, \ldots.. f_n$ are the frequencies of the variable values $X_1, X_2, X_3, \ldots\ldots X_n$ respectively, then the weighted arithmetic is given by

$$\overline{X}_W = \frac{W_1(fX_1) + W_2(fX_2) + W_3(fX_3) + \ldots.. + W_n(fX_n)}{W_1 + W_2 + \ldots.. + W_n}$$

Where, $W_1, W_2, W_3, \ldots\ldots W_n$ are the respective weights of $X_1, X_2, X_3, \ldots\ldots X_n$

Example

Lets take another example: The average marks of 30 students in a section of class X are 20 while that of 20 students of second section is 30. Find the average marks for the entire class X?

Sol. We can do the question by using both the Simple average & weighted average method.

Simple average = $\frac{\text{Sum of marks of all students}}{\text{Total number of students}} = \frac{20 \times 30 + 30 \times 20}{30 + 20} = 24.$

By the weighted mean method, Average = $\frac{2}{5} \times 30 + \frac{3}{5} \times 20 = 12 + 12 = 24.$

Why Average changes

We know **Simple Avg. = Sum/Number**

So the average will change only on account of change in Sum or Number or Both.

Continuing with the simple example of first five natural numbers; we add a new quantity 9 to the available five numbers. Now the average will change on account of both the sum & the number of observations changing.

So, New Average = $\frac{15 + 9}{6} = 4.$

TIP

The mean of odd consecutive numbers is always the middle value etc.

This can also be calculated as New Average = 3 + 6/6 = 4.

Because '9' being greater than the present average by 6, the new average will be higher than the initial average. Find for yourself what will be the average when instead of 9, - 9 is added to the first five natural numbers?

Again coming back to the original 5 natural numbers case we replace 5 by 10. Now the number of quantities remains 5 but the sum has been altered, so the average will change. New Average = $\frac{15-5+10}{5}$ = 4.

This can also be calculated as: 3 + {(10 – 5)/5} = 4.

Toolkit

If each no. is increased / decreased by a certain quantity n, then the mean also increases or decreases by the same quantity.

e.g. New $\bar{x} = \frac{\sum_{i=1}^{n}(x_i + a)}{n} = \frac{\sum x_i}{n} + \frac{\sum a}{n} = \bar{x} + \frac{na}{n} = \bar{x} + a$

If each no is multiplied/ divided by a certain quantity n, then the mean also gets multiplied or divided by the same quantity.

e.g. New $\bar{x} = \frac{\sum_{i=1}^{n}\frac{x_i}{a}}{n} = \frac{\frac{1}{a}\sum_{i=1}^{n}x_i}{n} = \frac{\bar{x}}{a}$

Average Speed

Average Speed = $\frac{\text{Total distance covered}}{\text{Total time taken}}$

If x_1 & x_2 are the distances covered at velocities v_1 & v_2 respectively then the average velocity over the entire distance $(x_1 + x_2)$ is given by

$$\frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{x_1 + x_2}{\frac{x_1}{v_1} + \frac{x_2}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}.$$

TIP

The average velocity is **NOT** = Sum of the two velocities/2.

Example

The average weight of 30 students in a class is 60 kg. When 20 new students are admitted, the average weight is reduced by 2 kg. Find the average weight of the new students.

Sol. **<u>Let's try by a Simple logical method:</u>**

Here, 20 new students are admitted and the final average decreases. If we assume that the average weight of all the new members be 60 kg, then the average weight of all the 50 members would have been 60 kg. But weight of each member has decreased by 2 kg.

⇒ Sum of weight of 20 new students admitted = (20 × 60) + (– 2 × 50) = 1,100kg.

⇒ Average weight of 20 new students admitted = $\frac{1100}{20}$ = 55 kg.

The captain of a cricket team of 11 players is 25 years old and the wicket keeper is 3 years older. If another two players replace these two players, the average of the cricket team drops by 2 years. Find the average age of these two players.

Sol. Let's say the average age of the whole cricket team was x years, and the average age of the two replacing players is y years.

Then according to the question:

$\frac{11x - 25 - 28 + 2y}{11} = x - 2$

11x – 53 + 2y = 11x – 22

2y = 53 – 22 ⇒ 31

y = $\frac{31}{2}$ ⇒ 15.5 years.